Separated Bubble Induced in the Laminar Boundary Layer by a Slight Depression in the Wall

B. Prunet-Foch,* F. Legay-Desesquelles,† and G. B. Diep‡
Laboratoire d'Aérothermique du C.N.R.S., Meudon, France

The existence on the surface of a flat plate of a slight hollow deformation usually will result in the creation of an elongated separated bubble. The theoretical analysis of the phenomenon is based on the integral method of Dorodnitsyn, with the concept of free interaction and, for the gradient of the velocity profile in the boundary layer, the expressions suggested by Nielsen. That enables one to complete the calculation of all of the separation zone up to the reattachment point. In the zone of expansion of the flow preceding separation, where the Nielsen theory does not apply, the calculation has been made by solving the boundary-layer equations by a finite-difference method. The characteristics of the flow thus have been computed in the entire domain of interest. The results of computation generally agree fairly accurately with the experimental measurements. This has enabled us to express laws governing the behavior of the bubble as a function of the physical data of the hollow and the main flow.

Nomenclature

a	= amplitude
c_i	= functions specifying the velocity profile of the outer flow, $i=1, 4$ [Eq. (1)]
C_f, C_p	= local coefficients of skin friction and pressure at the wall
l	= reference length, taken equal to x_0 herein
M	= Mach number
Re	= Reynolds number
u	= axial velocity in x, y plane
$\bar{u}$	= u/u_e
U_0	= u_0
x, y	= coordinates of physical flow under consideration
x_s	= value of x at the beginning of the sine-wave hollow
X_{rea}	= value of x at reattachment point
X_{sep}	= value of x at separation point
X, Y	= coordinate of Stewartson plane
α_W	= value of $\partial\bar{u}/\partial\eta$ at the wall
γ	= ratio of specific heats
δ_l	= displacement thickness
λ	= wavelength
ν	= kinematic viscosity of the fluid
ξ, η	= coordinate of Dorodnitsyn plane

Subscripts

e	= value at the outer edge of the boundary layer
G	= value at the boundary between the two domains
0	= initial value
S	= value on the line $S(x)$
w	= wall conditions
∞	= freestream conditions

Introduction

THE accurate prediction of the phenomenon of separation in the boundary layer still remains an important problem in fluid dynamic research. During recent years, several theories and experiments have taken place in that field. Studies have been made in cases of surface deformation of sine-wave shape with small amplitude. Inger,[1] among others, has given a method of calculating the characteristics of a subsonic laminar flow and Inger and Williams[2] did so for turbulent flows. In such cases, as has been shown by Hung and Fan,[3] the method is applicable only in a region very near the surface; but, in any case, as far as turbulent flows are concerned, separation does not occur in the case of such small surface deformation.

Recently, Polack et al.[4] have shown that it is possible to solve the problem for a supersonic laminar boundary-layer flow along the sine-wave wall sections, beginning by a protruding portion. To solve the boundary-layer equations written in terms of the Levy-Lees variables, they use an alternating-direction-implicit method. They give the local skin-friction coefficient and the distribution of the heat transfer to the wall. The present study relates to the supersonic laminar separation followed by reattachment, which may occur in a single slight hollow deformation situated at a given distance from the leading edge of a flat plate.[5]

Dynamic Study

General Equations

The flow along a smooth flat plate in which there is a hollow of small amplitude at a given distance from the leading edge may be considered, in a first approximation, to be governed by steady two-dimensional boundary-layer equations. The complete investigation of the problem, separated zone included, leads one to distinguish within the flow two domains, I and II, in which the boundary-layer equations have to be solved by two different methods (Fig. 1).

In domain I, which is composed of the flow along the plane portion and the beginning of the hollow, the boundary-layer equations are solved by a finite-difference method. This inter alia supplies the velocity profile required to initiate the calculation in domain II starting from abscissa x_G.

In domain II, the complete phenomenon of separation and reattachment can be predicted by a computation based on the Dorodnitsyn method.[6] Adopting the concept of free interaction and Nielsen's expression[7] as regards the repartition of velocity in the boundary layer, it is possible to obtain the

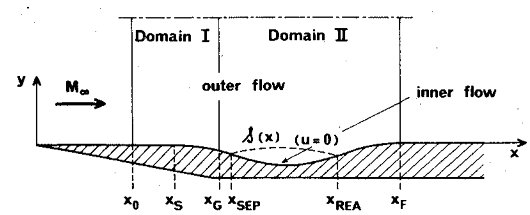

Fig. 1 Domains to which calculation applies.

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*Attaché de Recherche.
†Ingénieur de Recherche.
‡Maître de Recherche.

dynamic field in the neighborhood of separation and in all of the separated zone.

Calculation in Domain I

The Stewartson transformation[8] reduces the general equations of the boundary layer, for a compressible flow, to those of an incompressible flow. To solve these equations by the method of finite difference, it is sufficient to know the following in the Stewartson plane: 1) the expression of the potential flow along the wall which is obtained by the small-perturbation theory, and 2) the distribution of velocities in the boundary layer at two stations $x_0-\Delta x$ and x_0, x_0 being the abscissa at which the calculation begins.

In cases selected for experimental comparisons, the results are improved by a displacement thickness correction to the boundary-layer solution. This pushes the location of the separation point upstream and leads to better agreement with the experimental locations. Indeed, coming to the vicinity of the separation, a substantial increase of the thickness of the boundary layer takes place; therefore the displacement thickness δ_1 increases rapidly. That correction is based upon the following assumption. The actual flow outside the boundary layer behaves in the same manner as the one that occurs on a fictitious surface situated at the distance δ_1 above the actual plate, δ_1 then being computed according to the first calculation made without correction. The expansion law of Prandtl-Meyer is applied again to obtain the variation of the main flow along that surface.

Calculation in Domain II by Means of Nielsen Method

In domain II, the characteristics of the flow are obtained by the Nielsen method. The expression utilized to determine the velocity profiles before separation is

$$\frac{\partial \bar{u}}{\partial \eta}=\frac{(1-\bar{u})\sqrt{\bar{u}+c_4}}{c_1+c_2\bar{u}+c_3\bar{u}^2+\ldots} \tag{1}$$

Nielsen, applying Dorodnitsyn's method, reduces the boundary-layer equations as obtained through the Stewartson transformation to an integrodifferential equation by introducing smoothing functions in the form $f(\bar{u})=(1-\bar{u})^n$ or $f(\bar{u})=\bar{u}^n\ (1-\bar{u})$. That equation, together with that of free interaction,[6] then supplies the equations of the differential system, enabling computation of c_i as functions of x. Separation and reattachment occur when c_4 becomes zero.

After the separation, a reverse flow appears. Nielsen then considers in the boundary layer two regions separated by the line $\mathcal{S}(x)$, where $\bar{u}$ is zero. In the inner region, that is, between the wall and the line $\mathcal{S}(x)$, he chose for the velocity profile a polynomial in η of arbitrary degree. In the outer region, on the other hand, the expressions used before separation are taken again.

Passing From Domain I to Domain II

The method of computation of the dynamic field inside domain II thus being adopted, it remains only to define the initial values of functions c_i at the boundary G where that region starts. To compute these, the velocity profiles obtained at the end of the calculations by the finite-difference method in domain I (i.e., at abscissa x_G) are used. Selecting four values η_i of η arbitrarily, this profile supplies the values of velocities $\bar{u}_i$ and the slopes $\partial\bar{u}/\partial\eta$ at the related points. Those quantities, once inserted in Eq. (1), lead to a system of four equations with four unknown quantities c_1, c_2, c_3, c_4.

The solution of that nonlinear system is obtained easily if one of the selected values corresponds to $\eta=0$, which allows the value of friction at the wall to remain unmodified. In that case, the corresponding equation reduces to

$$\left.\frac{\partial \bar{u}}{\partial \eta}\right|_w=\frac{\sqrt{c_4}}{c_1}=\left(\frac{U_0}{U_e}\right)^2\ell\sqrt{\frac{U_0\ell}{\nu_0}}\left.\frac{\mathrm{d}U}{\mathrm{d}y}\right|_w \tag{2}$$

Under these conditions, one obtains an implicit equation in c_1 which is solved by an iteration procedure. The values of c_2, c_3, and c_4 then are obtained directly in terms of c_1.

Initial Velocity Profiles

Having so explained the calculation methods used inside the two regions of the flow and how to pass from one to the other, it remains to indicate precisely how to obtain the initial velocity profiles. These profiles, which are necessary for solving the equations by a finite-difference method, could have been obtained, as can any profile on a flat plate, through any of the classical methods, for instance that of Chapman and Rubesin.[10] In order to keep the number of calculation processes employed in this study to a strict minimum, we have thought it suitable to resort once again to the method of Nielsen. As the way to determine these velocity profiles, we refer, therefore, to the Nielsen method, as explained in Ref. 7. When dealing with a flat plate, the relation of similitude of the velocity profiles leads to a simplification of the differential system, and the result actually is obtained by solving an algebraic system of equations.

Experiments, Calculations, and Comparison of Results

Experimental Apparatus

Measurements have been made in the supersonic wind tunnel of Aérothermique Laboratory. The Mach number M_∞ is predetermined to be 1.92 for the flow that we shall call *F1*, and 2.41 for the flow designated by *F2*, the corresponding Reynolds number *Re* being 13×10^6/m and 9.7×10^6/m. The models are flat plates with a hollow of sine-wave shape (amplitude $a=0.3$ or 0.4 mm, wavelength $\lambda=30$ or 40 mm). The hollow is situated at a distance $x_s=20$ or 30 mm from the leading edge. Those models are fitted with static pressure taps to measure the distribution of static pressure at various distances.

The thickness of the boundary layers obtained in the wind tunnel is on an average smaller than 1 mm, which necessitates scanning by means of miniaturized total pressure probes built at the laboratory. The flow can be visualized either at the level of the plate surface, through the deposit of a paraffin film, or as a whole, using a schlieren apparatus. Both methods supply an approximate value of the separation point.

Sequence of Calculations

The calculation program enables one to obtain the characteristics of the separated bubble induced by the sine-wave hollow in a flat plate. The data are as follows: 1) at infinity upstream: Mach number, pressure, and temperature; and 2) on the plate: the equation of the wall section.

The calculation can be performed on the condition that the expression of the potential flow is known up to the abscissa, where the separation occurs. That expression can be obtained, for instance, from the small-perturbation theory. In the application considered in this study, we have chosen sine-wave sections defined by

$$y=-a+a\cos[(2\pi/\lambda)(x-x_s)] \tag{3}$$

The separation abscissa is found by applying solely the finite-difference method. Then the value of x_G is adjusted by successive computation to arrive, in domain II, at a separation abscissa equal to that just obtained. Another adjustment applying to the η_i also is made in order to have a good coincidence of the profiles when passing from one domain to the other.

The calculation then may be pursued in the entire separated zone up to reattachment. In all of this domain, at each abscissa, the values of c_i, U_e, η_s, and α_w $[\alpha_w(=\partial\bar{u}/\partial\eta)_w]$ thus are found. The time to complete the computation evidently depends on the step sizes chosen in the boundary-layer calculation; taking steps of $\Delta x=0.03$ mm, $\Delta y=0.004$ mm,

Table 1 Results from placing plates in flow *F1*

	x_s, mm	x_{sep}, mm Experiment	x_{sep}, mm Calculation	x_{rea}, mm Experiment	x_{rea}, mm Calculation
H1	30	38-39	39.4	≈47.5	48.4
H2	20	32-33	33.6	≈44	44.8

and 100 points for the velocity profiles, for a wavelength of 4 cm, the time of calculation on an IBM 370/165 never has exceeded 2.5 min.

Beyond the reattachment point, a new potential flow solution should be used on conjunction with the solution of the boundary-layer equations by finite-difference methods. However, at the given experimental conditions, beyond reattachment the boundary layer is no longer laminar, and the calculation is no longer valid.

Comparison of Results

Numerous experiments have been carried out, and the results have been compared with those obtained through calculations made for the same flow conditions and the same wall section.

Separation and Reattachment Abscissae

The comparisons show that the method of finite difference gives a separation abscissa x_{sep} very close to the one given by experiments. If, however, the calculation predicts a separation occurring slightly further than the experiment, such slight discrepancy can be accounted for by the fact that even a very thin leading edge induces a shock wave in the flow; the disturbance caused by such a shock wave may influence the beginning of the separation process. The separated flow occurs within the hollow. It reattaches toward the end of the deformation of the wall at an abscissa x_{rea}. Qualitatively, the results show that, the sooner the separation occurs, the bigger will be the bubble and the further off reattachment.

H1 and *H2* are two different sine-wave hollows of "amplitude/wavelength" ratio equal to 1/100, the respective amplitudes being 0.3 and 0.4 mm and the corresponding distances from the leading edge being 30 and 20 mm. The results from placing such plates in a flow *F1* are shown in Table 1. In the case of a hollow of type *H2* situated 30 mm from the leading edge, we obtain, for a flow *F2*, an abscissa of separation $x_{sep} = 42.7$ mm, reattachment occurring at $x_{rea} = 56.7$ mm.

Dynamic Behavior of the Flow

Static pressure measurements at the wall give the variations of the pressure coefficient, $C_p = (2/\gamma M_\infty^2)[(p/p_\infty) - 1]$, with the distance from the leading edge. These variations agree well with the results from the calculation (Fig. 2). We note that an inner velocity profile of the third degree rather than of the second degree improves the agreement. However, a second-degree profile yields satisfactory accuracy. The slight discrepancy that is found can be explained by the effect of the leading-edge shock. As an example of the effect of the leading edge, Fig. 3 shows the variations of C_p for an identical hollow *H2* situated 20 mm in a flow *F2*, as opposed to the preceding figure, where $x_3 = 30$ mm and the main flow is *F1*. The discrepancy between measured points and points as calculated is more apparent. But if, when using the experimental measurement, one had taken the pressure as measured on the flat portion of the plate (i.e., downstream of the shock wave) instead of the static pressure at infinity, the points calculated and those found experimentally then would coincide (a simple translation of the entire curve to the position shown by the dotted line).

Referring again to Fig. 2, we have drawn in a dotted line the variation of the pressure coefficient such as would be obtained if separation did not occur. Thus one can see that separation tends to reduce the amplitude of variation of the coefficient C_p. If it is not possible to effect a quantitative comparison of the values obtained directly, it still remains that the general shape of the variation as calculated is very close to that given by Polak et al. (Ref. 4, Fig. 5c).

The velocity profiles measured and calculated also coincide satisfactorily (Fig. 4); consequently, the friction coefficient C_f at the wall which has been computed is close to the values that would be inferred from the experimental velocity profiles (Fig. 5). We find the same type of variation as has been found by Polak et al. (Ref. 4, Fig. 4).

On Fig. 4, in the zone immediately preceding reattachment (see velocity distribution $x = 55$ mm), we note a discrepancy between experiment and theory. This discrepancy may be explained by the fact that the separated bubble provokes an earlier transition. That transition begins before the reattachment. The experimental velocity profile at abscissa $x = 60$ mm shows clearly that the flow after reattachment no longer is laminar.

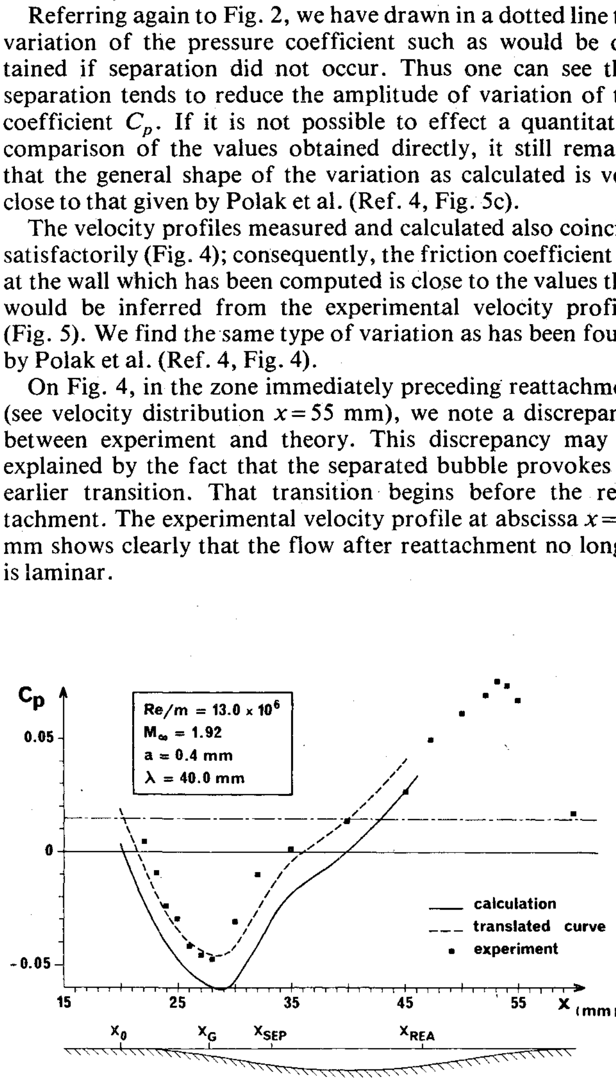

Fig. 3 Variation of pressure coefficient.

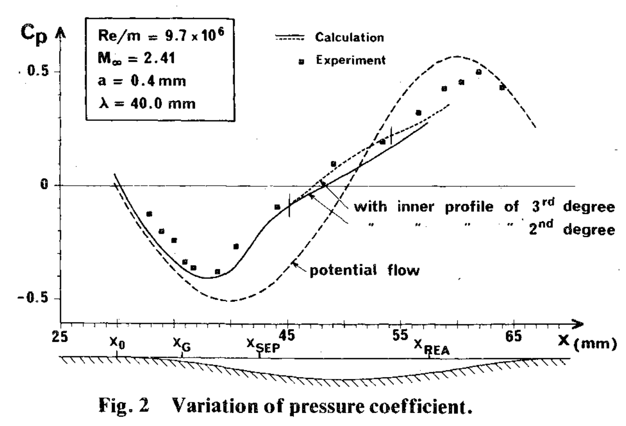

Fig. 2 Variation of pressure coefficient.

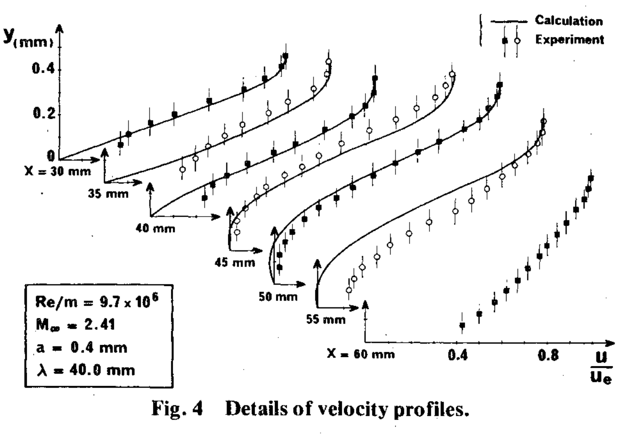

Fig. 4 Details of velocity profiles.

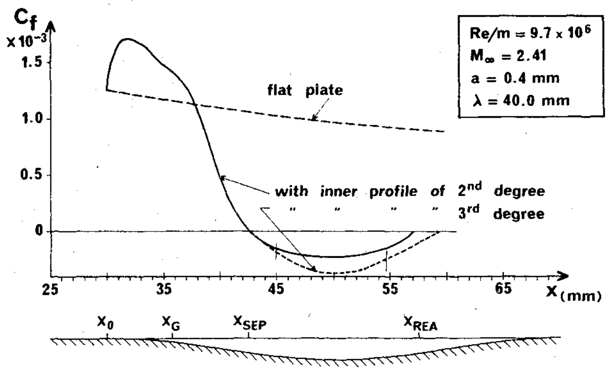

Fig. 5 Variation of local skin friction.

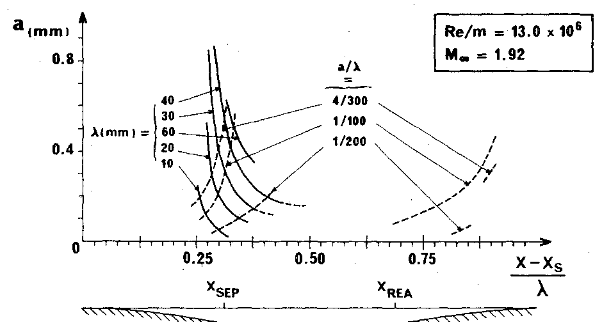

Fig. 6 Variation of separation and reattachment abscissae in relation to *a* and λ.

Application of Calculation to More General Cases

The good agreement found between the results of experiments and those of calculation allows the use of this calculation procedure in different cases. Thus, for a given distance of the beginning of the hollow from the leading edge, some laws governing the behavior of the bubble can be set forth as follows. The main flow being given (Fig. 6), then 1) for a given wavelength the fraction $(x_{sep} - x_s)/\lambda$ increases when the amplitude of the sine-wave hollow decreases; 2) for each wavelength there exists a value of amplitude for which the calculation does not predict a separation, a value that appears to increase as wavelength increases; and 3) for a given amplitude, the wavelength increases as the value of the fraction $(x_{sep} - x_s)/\lambda$ increases.

Conclusion

The present study enables us to conclude that, for a supersonic flow, the existence of a hollow on the surface of a flat plate generally induces in a laminar boundary layer a flat elongated bubble, even in the case when the ratio of amplitude/wavelength is small. The flow will become reattached before the end of the hollow. The use of a finite-difference method in the expansion zone at the beginning of the hollow and of the Nielsen method in the zone thereafter leads to a computation of the dynamic field of the flow which agrees with results found experimentally in a wind tunnel, at least when the deformation hollow is of sine-wave shape.

The slight discrepancy found in the distribution of the local pressure coefficient may be explained by the fact that the actual flow at the leading edge differs from that assumed in the theory. Even though that phenomenon tends to induce an early separation, the methods applied here lead to a good knowledge of the flow. Consequently, the calculation supplies valid estimates of the local friction coefficient. The computed variation is comparable to that found by earlier investigators in the case of multiple sine-wave protuberances. The method employed in the separated zone can be improved by the use of a larger number of terms in the expression for velocity profiles; the computation then necessitates as many new equations as the number of new parameters thus introduced.

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